Converting Release-04 Gravity Coefficients into Maps of Equivalent Water Thickness

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This document describes the post-processing procedures that are used to convert Level-2 Release-04 (RL04) gravity field coefficients from the two central GRACE Science Data System (SDS) centers (The Center for Space Research (CSR) and GeoForschungsZentrum (GFZ)) as well as the validation center (Jet Propulsion Laboratory (JPL)) into maps of equivalent water thickness and ocean bottom pressure (OBP). The RL04 coefficients represent a significant improvement over previous releases of the data, but this document will not describe the specific processing changes. Readers who are interested in the details are advised to read the *Processing Standards* documents on the data archive site (ftp://podaac.jpl.nasa.gov/grace/doc). Here we will describe the changes made to the RL04 coefficients that are distributed on the Level-2 Data Archives as GSM files.

The GSM files contain Stokes coefficients representing the full gravity field. To make maps of change in land water storage and OBP (in terms of equivalent water thickness), we first compute the time-mean of the Stokes coefficients from January 2003 to December 2006 and remove it in order to determine the Stokes coefficient anomalies ($\Delta C_{lm}(t)$, $\Delta S_{lm}(t)$). This is done separately for the CSR, GFZ, and JPL solutions.

As first noted by *Swenson and Wahr* [2006], the coefficients from all centers suffer from a correlated error that causes "stripes" in the maps that are not fully removed by a simple Gaussian smoothing. Thus, it is necessary to "de-stripe" the coefficients before converting to maps. We use a modification of the algorithm described in *Chambers* [2006a]. In the RL04 data, we have found that several parameters in the filter can be relaxed and still provide maps without significant stripes. The new filter that has been implemented for RL04 coefficients keeps the lower 11x11 portion of the coefficients unchanged, as well as all order 0 and order 1 coefficients. A 5th order polynomial is fit as a function of even or odd degree (n) to the remaining coefficients for each order (m) greater than 2 from $n = 12$ (or $n = m$ if $m > 11$) up to $n = 60$. Only one polynomial is computed for each odd or even set for a given order. Only coefficients up to $m = 40$ are

de-striped. Every coefficient above n = 40, m = 40 is set to zero. The maximum order has been reduced from the filter in *Chambers* [2006a] because CSR_RL04 coefficients are only solved to n = 60, m = 60.

For example, consider de-striping the ΔC_{nm} coefficients for order 5. Since the lower 11x11 portion is kept unchanged, one would compute separate 5th-order polynomials as a function of n fit to $\Delta C_{12,5}$, $\Delta C_{14,5}$, $\Delta C_{16,5}$, … $\Delta C_{60,5}$ and $\Delta C_{13,5}$, $\Delta C_{15,5}$, $\Delta C_{17,5}$, … $\Delta C_{59,5}$. Then, the filtered coefficients ($\Delta\hat{C}_{nm}$) would be

$$\begin{aligned} \Delta\hat{C}_{12,5} &= \Delta C_{12,5} - fit_{even}(n=12);\ etc \\ \Delta\hat{C}_{13,5} &= \Delta C_{13,5} - fit_{odd}(n=13);\ etc \end{aligned} \quad . \tag{1}$$

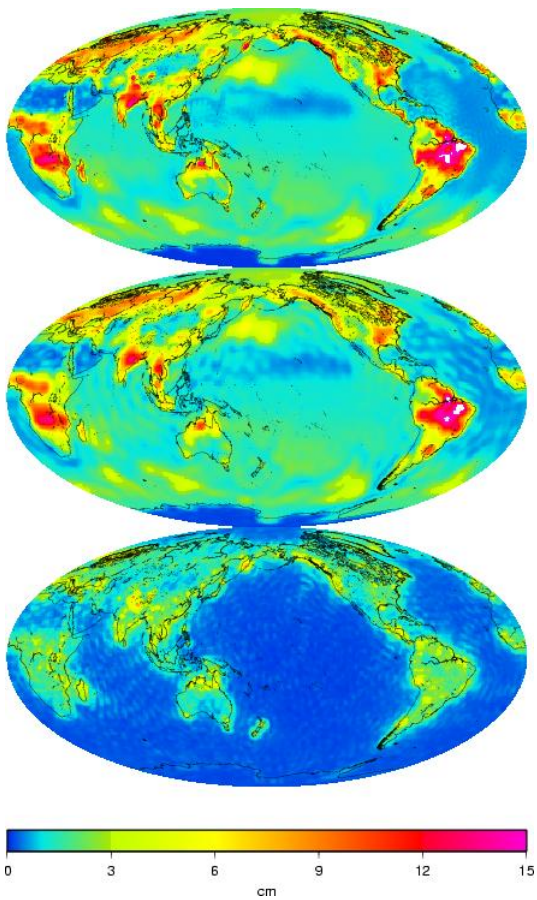

Figure 1. RMS of simulated water thickness grids for no truncation (top), truncated to degree/order 40 (middle), and RMS of difference (bottom).

Any filtering will remove signal. We test how much signal is removed with this de-striping algorithm by first creating a set of simulated gravity coefficients without stripes based on output from the GLDAS/NOAH hydrology model over the land [*Rodell et al.*, 2004] and the Ocean Model for Circulation and Tides (OMCT) over the ocean [*Thomas*, 2002]. Monthly output from January 2003 through December 2006 were used to compute RMS statistics. Truncating to degree 40 without de-striping removes a small amount of signal over the land (Figure 1), with only minimal signal removed over the ocean. The total RMS of the signal without truncation is 3.3 cm and decreases to 3.1 cm after truncating to degree 40. The maximum local error introduced by the truncation is about 3 cm RMS and is very localized over land, generally at the boundaries with the ocean (Figure 1, bottom). The average error introduced by truncation is about 1 cm RMS over land and less than 5 mm RMS over

the ocean.

After the de-striping algorithm is applied, an additional small amount of signal is removed (Figure 2), reducing the total RMS slightly to 3.0 cm, suggesting that the de-striping algorithm reduces the signal on average by less than 1 cm RMS in the ocean and 2 RMS over land, with approximately equal parts coming from truncation of the coefficients and the de-striping. If the GRACE coefficients are not de-striped, however, the signal is dominated by the "stripes" (Figure 3).

By smoothing the coefficients with a Gaussian of a certain radius [e.g., Wahr et al., 1998], the loss of signal due to the de-striping is attenuated. With an additional 300 km smoothing (Figure 4) the variability of the maps with de-striping is reduced from 2.74 cm RMS to 2.65 cm RMS, suggesting an average mean error of less than 1 cm. With an additional 500 km smoothing (Figure 5) the variability of the maps with de-striping is reduced from 2.46 cm RMS to 2.41 cm RMS, suggesting an average mean error of less than 0.6 cm.

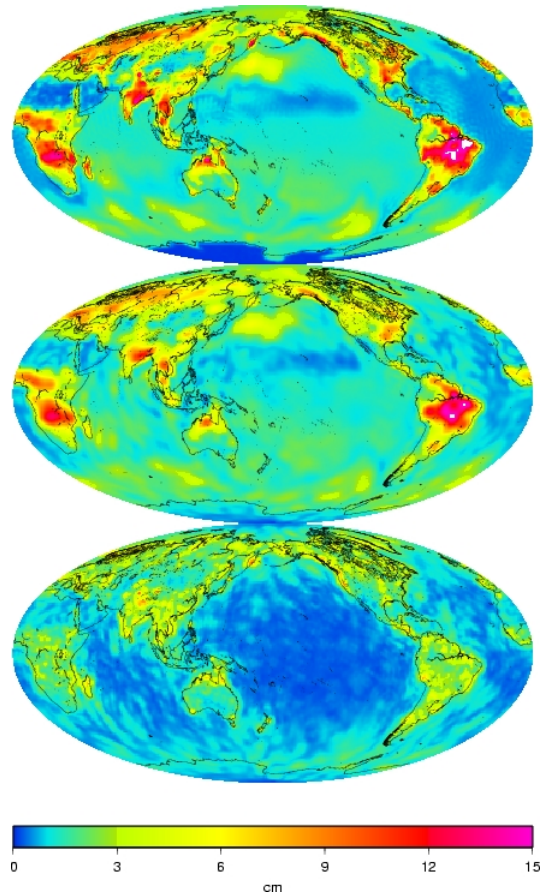

Figure 2. RMS of simulated water thickness grids without de-striping (top), after de-striping (middle), and RMS of difference (bottom).

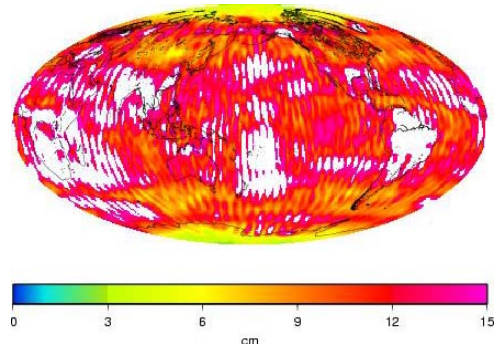

Figure 3. RMS of GRACE data without de-striping algorithm.

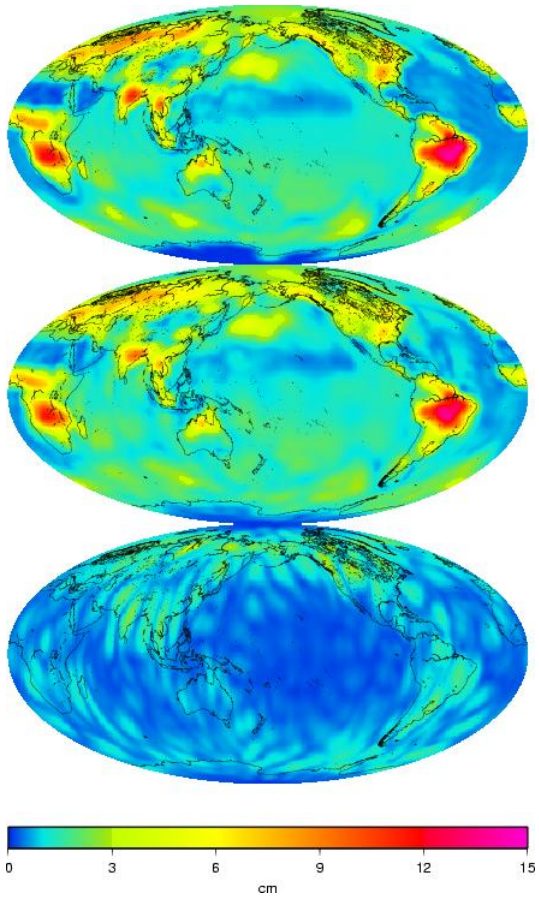

Figure 3. RMS of simulated water thickness grids truncated to degree/order 40 (top), after applying de-striping algorithm (middle), and RMS of difference (bottom). All are smoothed with a 300 km Gaussian.

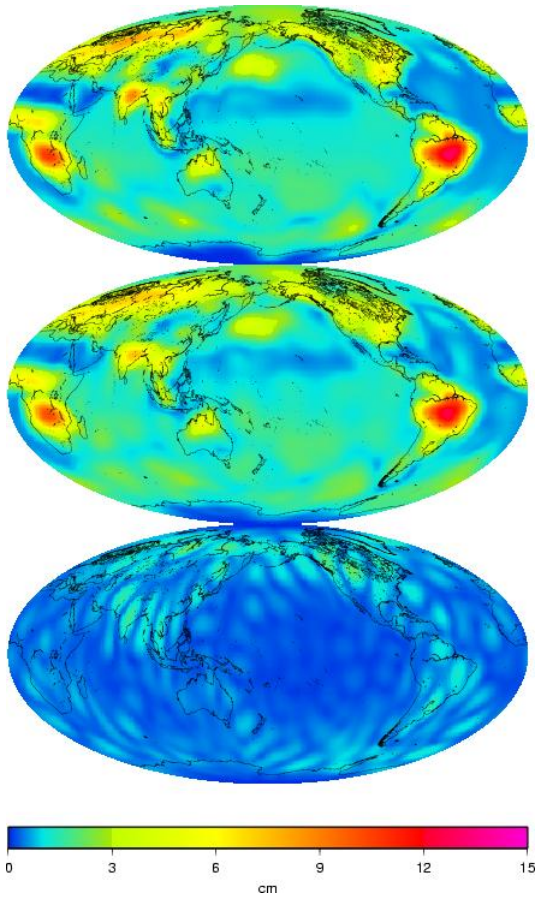

Figure 4. RMS of simulated water thickness grids truncated to degree/order 40 (top), after applying de-striping algorithm (middle), and RMS of difference (bottom). All are smoothed with a 500 km Gaussian.

After de-striping, the monthly degree 2, order 0 coefficients estimated with GRACE are replaced with those from a satellite laser ranging (SLR) analysis [*Cheng and Tapley*, 2004] after removing the effects of the same atmosphere/ocean model used in the GRACE processing and removing a similar time-average as used with the GRACE coefficients. This is done because it was noticed that there is a significant 162-day error in the GRACE degree 2, order 0 coefficients from all three SDS centers which appears to be related to an alias of the S_2 tide and strongly effects results over Antarctica and the tropical ocean [S. Bettadpur, personal communication, 2007].

As discussed in *Chambers* [2006b] and *Swenson et al*. [2007], the GRACE project does not provide degree 1 gravity field coefficients because of the reference frame used. However, to use GRACE data to study changes in water storage, one wants to translate the data to a reference frame where degree 1 coefficients are non-zero. Previously a mean, seasonal variations derived from SLR analysis have been used [e.g., *Chambers*, 2006b]. Here we use new, monthly geocenter estimates that have recently been calculated by *Swenson et al*. [2007], based on an ocean model and GRACE data. The new geocenter estimates are designed to model the non-atmospheric contribution to degree 1 fluctuations in the gravity field.

We have also applied a correction for glacial isostatic adjustment (GIA) based on the model by *Paulson et al*. [2007]. To GRACE, GIA causes a secular trend in the gravity field that is not due to the redistribution of water over the Earth's surface and must be removed. In order to do this, we first have to restore modeled rates for certain coefficients (degree 2, 3, and 4 for order 0, and degree 2, order 1) as discussed in the Processing Standards Documents [*Bettadpur*, 2007].

At this point, we have a set of gravity coefficient anomalies for each month ($\Delta\hat{C}_{nm}(t)$, $\Delta\hat{S}_{nm}(t)$) that have been "de-striped" and corrected for GIA, degree 1 terms, and for problems in the degree 2, order 0 term. We compute maps of water storage anomalies over the land ($\Delta\eta_{land}$) directly as

$$\Delta\eta_{land}(\phi,\lambda,t) = \frac{a_E\rho_E}{3\rho_W}\sum_{n=0}^{40}\sum_{m=0}^{n}\frac{(2n+1)}{(1+k_n)}W_n P_{nm}(\sin\phi)\left\{\Delta\hat{C}_{nm}(t)\cos m\lambda + \Delta\hat{S}_{nm}(t)\sin m\lambda\right\} \qquad (2)$$

where

$$W_n = \exp\left[-\frac{(nr/a_E)^2}{4\ln(2)}\right], \qquad (3)$$

and ρ_E is the average density of the Earth (5517 kg m^{-3}), ρ_W is the density of fresh water (1000 kg m^{-3}), a_E is the mean equatorial radius of the Earth, ϕ is the geographic latitude, λ is the longitude, $P_{nm}(\sin\phi)$ are the fully-normalized Associated Legendre Polynomials of degree n and order m, r is the Gaussian averaging radius, and k_n are load Love numbers of degree n.

One further step has to be made in order to obtain ocean bottom pressure. As discussed more completely in *Chambers* [2006b], the coefficients distributed by the GRACE project are deviations from a background ocean model. Thus, one has to add back the monthly OBP modeled in the GRACE processing [*Flechtner*, 2007]. A new OBP product (GAD) is now available from the project [*Flechtner*, 2007]. It gives the mean monthly gravity coefficients of the OBP only with no variations over land and includes spherical harmonics from degree 0 and higher. In order to restore the full model OBP, one needs to restore all GAD coefficients (including degree 0 and degree 1) to the GRACE coefficients, not just degree 2 and higher. This should have been done in the earlier ocean analysis [e.g., *Chambers*, 2006a; 2006b], but the appropriate files were not available. Thus to calculate OBP in terms of equivalent sea level, one would use

$$\Delta\eta_{ocean}(\phi,\lambda,t)=\frac{a_E\rho_E}{3\rho_W}\sum_{n=0}^{40}\sum_{m=0}^{n}\frac{(2n+1)}{(1+k_n)}W_nP_{nm}(\sin\phi)\left\{\begin{array}{l}\left(\Delta\hat{C}_{nm}(t)+\Delta C_{nm}^{GAD}(t)\right)\cos m\lambda+\\ \left(\Delta\hat{S}_{nm}(t)+\Delta S_{nm}^{GAD}(t)\right)\sin m\lambda\end{array}\right\}. \quad (4)$$

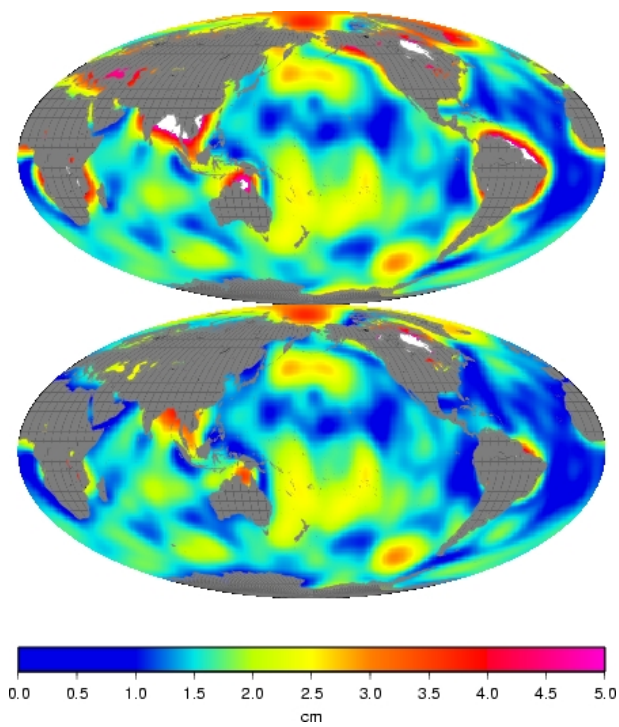

Figure 4. RMS of OBP from GRACE with a smoothing radius of 750km before (top) and after (bottom) removing leaked hydrology signals.

The vaiations in OBP are approximately five to ten times smaller than the water storage fluctuations over land. Because of this, hydrology signal can "leak" into the coastal waters, implying much larger OBP variations than really exist (Figure 4). *Wahr et al.* [1998] proposed a method to reduce this, using GRACE observations over land. As they noted, this is arguably better than using a hydology model, since most do not include all the loads of snow or ice. To do this, one has to first map GRACE coefficients into water storage anomalies, mask out the ocean areas, then de-compose the maps back into gravitational spherical harmonics. If these new "land-only" coefficients are then mapped into water thickness, the values over the ocean represent, to first order, the leaked signal from land. We followed this procedure using maps of water storage computed from the de-striped

GRACE coefficients with no additional smoothing. When the "land-only" coefficients are then smoothed and removed from the comparable GRACE OBP grids, the high variability around coastlines is largely eliminated (Figure 4).

In order to estimate the accuracy of the GRACE grids, the OBP is compared to the output from a version of the MIT general circulation model [*Marshall et al*., 1997] that is run at Jet Propulsion Laboratory (JPL) as part of the Estimating the Circulation and Climate of the Ocean (ECCO) consortium. We use comparisons over the ocean as an estimate of the error in the GRACE maps since the signal-to-noise ratio is so low here.

JPL_ECCO is a baroclinic model forced by winds, pressure, and heat and freshwater fluxes from the National Center for Environmental Prediction (NCEP) operational analyses products [*Fukumori et al*., 1999]. We use the simulation version that does not assimilate altimetry data because it extends over a longer time-period coincident with GRACE period we are studying. The JPL_ECCO model extends only between ± 78° latitude, so does not model barotropic fluctuations in the Arctic Ocean or near Antarctica. Since JPL_ECCO does not model the eustatic fluctuation in OBP caused by the exchange of water mass among the land, ocean, and atmosphere which GRACE will measure [e.g., *Chambers et al.*, 2004], the GRACE observations of mean ocean mass are added to the ECCO grids before computing statistics.

The RMS of the differences is computed over all available months for various smoothing radii (Table 1). Although statistics are only shown for CSR_RL04, the statistics for GFZ_RL04 and JPL_RL04 are essentially identical. These values represent the upper bound of uncertainty in the GRACE maps of equivalent water thickness for each smoothing radius.

Table 1. RMS of residuals of GRACE and JPL_ECCO OBP in cm for 39 months with different smoothing functions applied to both GRACE and JPL_ECCO. GRACE data are from CSR_RL04.

Smoothing Radius	**Mean RMS**
300 km Gaussian	2.3 cm
500 km Gaussian	1.7 cm
750 km Gaussian	1.4 cm

5. REFERENCES

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